Beam Splitters
and Single Photon Sources

PROJECT PRESENTATION

EE683A
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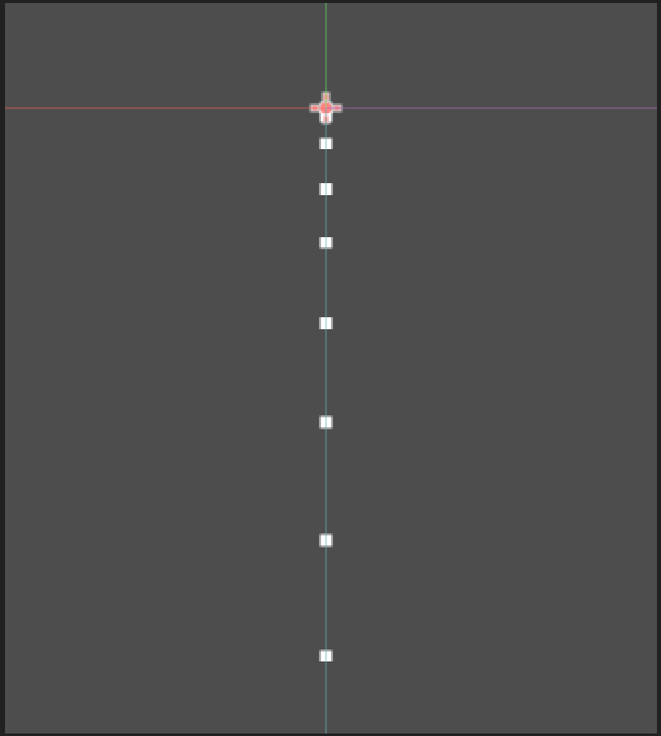
SINGLE PHOTON SOURCE

- A light source which emits light as single photons.
- The frequency of photons emitted can be time dependent.
- We have modeled three types of distributions
  a. Uniform
  b. Exponential
  c. Logarithmic



Source: Google Images

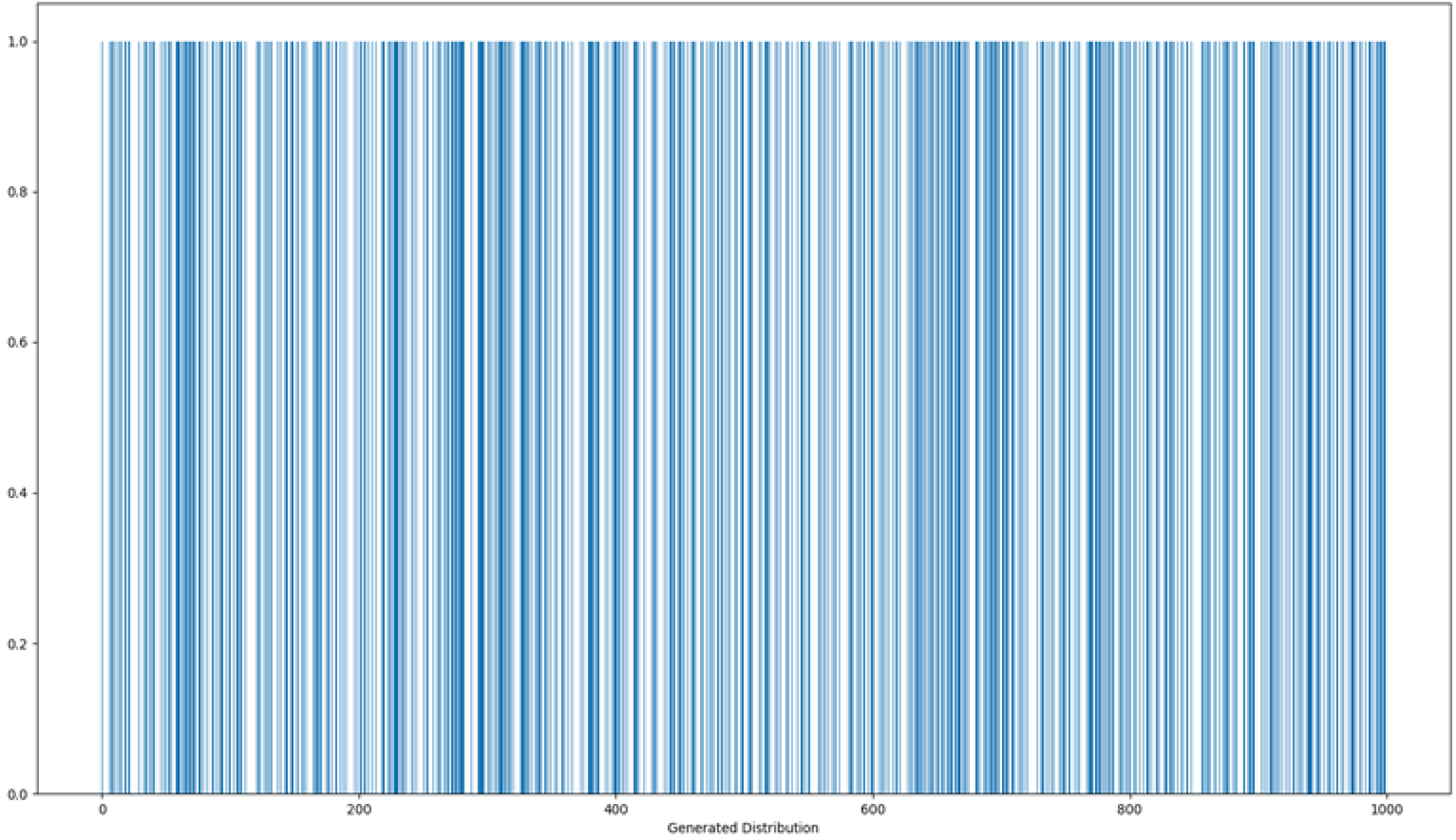
IMPLEMENTATION

- We used a **random number generator**
- Picks a number between 0 to 1 from a uniform distribution
  - Works for the uniform distribution
- For the other distributions, probability is time dependent
  - Our time-axis is the **length** of the array
- Set probability of getting 1 for each instant
  - Toss a biased ‘coin’ with above probability of getting 1
  - If you get 1, pick a random number between 0.5 and 1
  - If you get 0, pick a random number between 0 and 0.5
  - Above randomization is destroyed by **digitization**
- Digitization - discretize distribution using a threshold
  - We used a threshold of 0.5
  - Values above 0.5 become 1, others become 0
  - Threshold value affects total probability of getting a 1

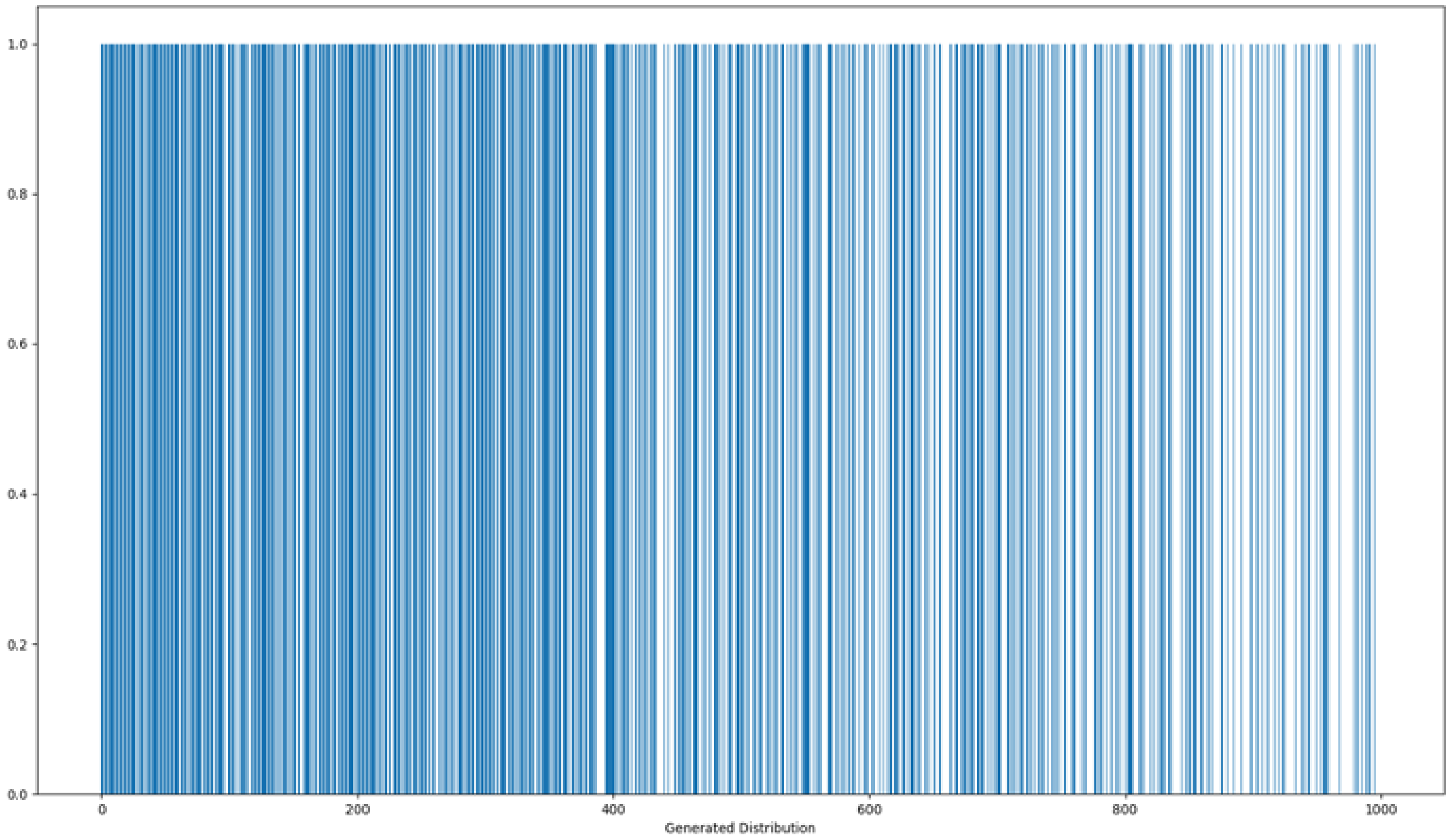
numpy.random.uniform

numpy.random.choice

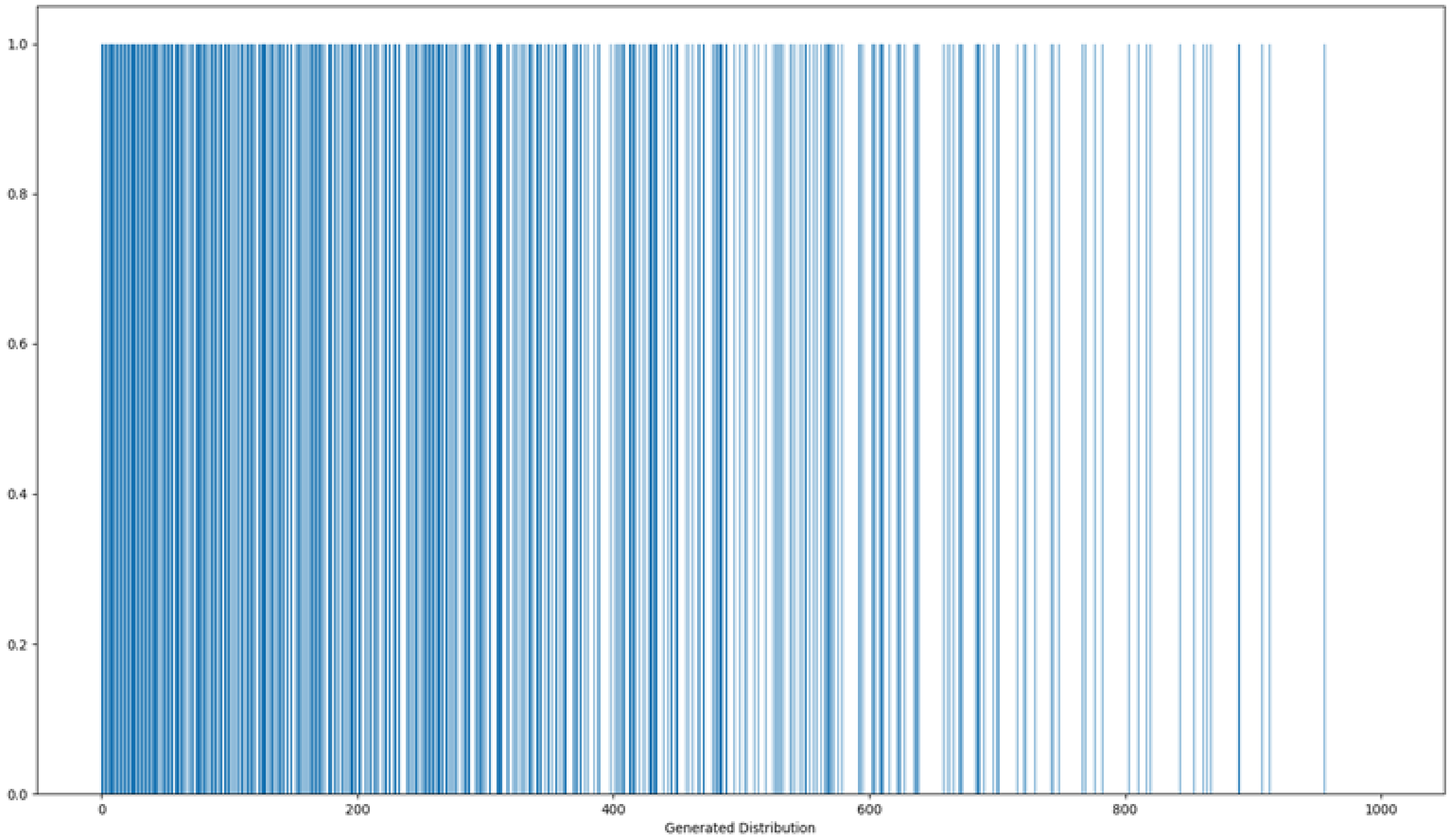
UNIFORM DISTRIBUTION



EXPONENTIAL DISTRIBUTION



LOGARITHMIC DISTRIBUTION



PROBLEM

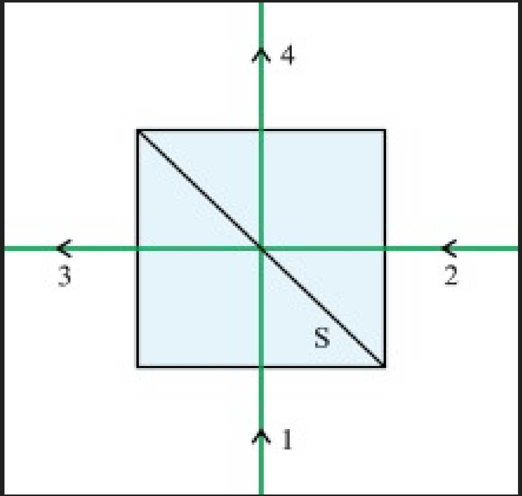
How do you differentiate an SPS from other light sources?

THE SOLUTION

You can split waves, but you can’t split photons.

BEAM SPLITTER

- Splits beams
- Can take two inputs
- Combines the two inputs to give two outputs
- Define transmittance and reflectance
    - Transmittance - probability that input beam passes through
    - Reflectance - probability that input beam gets reflected
    - Evidently sum to 1
- For a 50-50 beam splitter, reflectance is 0.5
- We assume our splitter to be **lossless**

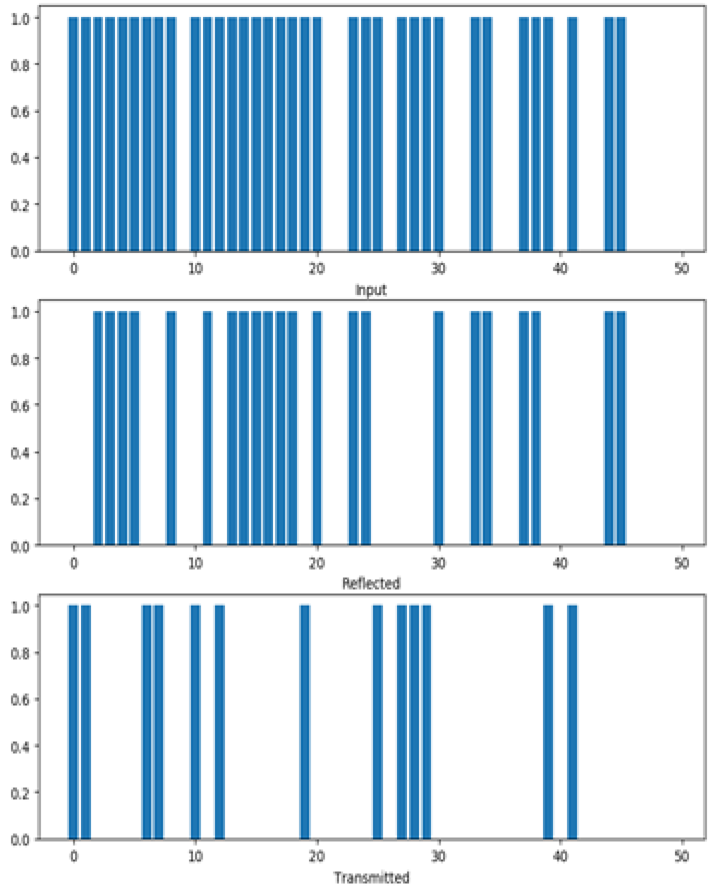


Source: [Google Images]

IMPLEMENTATION

- Again, we used an RNG
  - Picks a random number between 0 and 1
  - From a uniform distribution
- Set threshold equal to specified reflectance
  - If value less than threshold, send input to 'reflect' output
  - If value greater than threshold, send input to 'transmit' output
- Set output of the SPS as one input. Note:
  - At any instant only one of the two outputs can have a 1
  - This is called **antibunching**
  - SPS must show antibunching - photon cannot be at both outputs at the same time

INPUT - EXPONENTIAL DISTRIBUTION



CROSS-CORRELATION

$$(f \star g)[n] \triangleq \sum_{m=-\infty}^{\infty} \overline{f[m]}g[m+n]$$

- Sliding dot product of two functions - just like convolution
- Acts as a measure of similarity, of overlap between two functions
- Its value is directly proportional to the amount of overlap

- The output photon streams can be treated as electromagnetic waves
- We define **first-order correlation** as cross-correlation between the electric field amplitudes

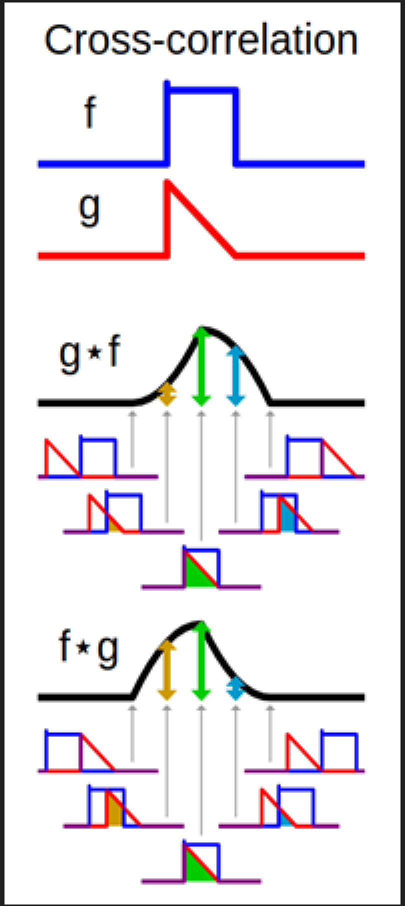
$$g^{(1)}(\tau) = \frac{\langle E^*(t)E(t+\tau)\rangle}{\langle |E(t)|^2 \rangle}$$

- Then we define **second-order correlation** as cross-correlation between the intensities of the waves

$$g^{(2)}(\tau) = \frac{\langle I_3(t)I_4(t+\tau)\rangle}{\langle I_3(t)\rangle\langle I_4(t+\tau)\rangle}$$

For an SPS we expect $g^{(2)}(0) = 0$ as at any instant the photon cannot be at both outputs

ANTIBUNCHING: $g^{(2)}(\tau) > g^{(2)}(0)$



Source: [Wikipedia]

CALCULATING CORRELATION

- Intensity is directly proportional to square of electric field
- Using $E(t) = E_0(ae^{i\phi} + a^\dagger e^{-i\phi})$ we get

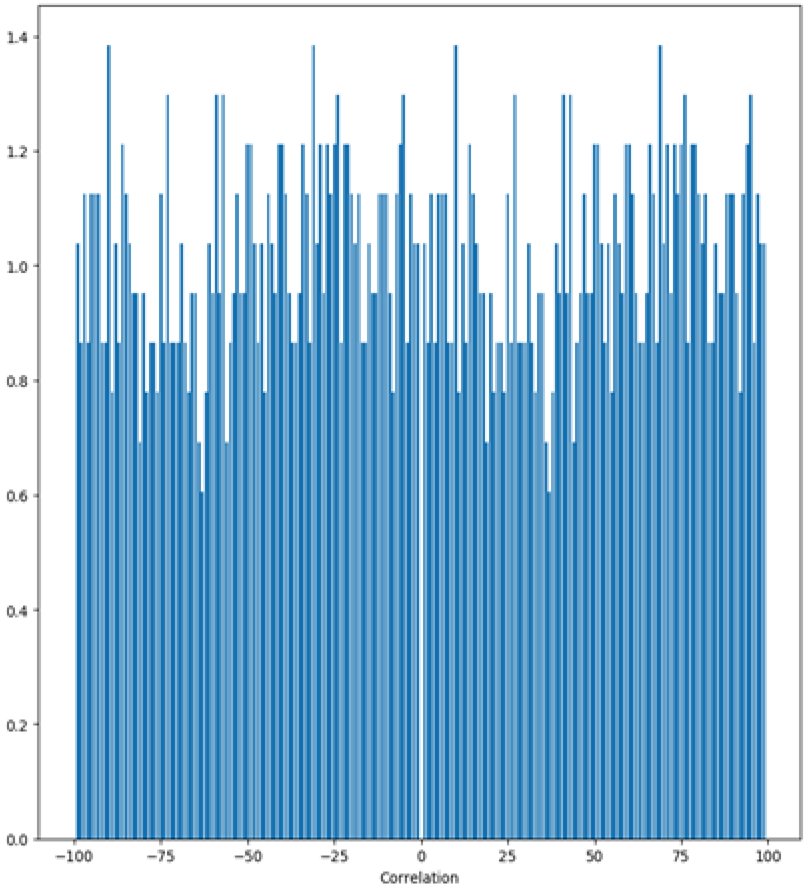
$$g^{(2)} = \frac{\left\langle \hat{a}^\dagger(t)\hat{a}^\dagger(t+\tau)\hat{a}(t+\tau)\hat{a}(t)\right\rangle}{\left\langle \hat{a}^\dagger(t)\hat{a}(t)\right\rangle^2}$$

$$g^{(2)}(\tau) = \frac{<N_3(t)N_4(t+\tau)>}{<N_3(t)><N_4(t+\tau)>} = \frac{<p_3(t)p_4(t+\tau)>}{<p_3(t)><p_4(t+\tau)>}$$

- The number distributions are what we get as outputs - put them in

$$(f \star g)[n] \triangleq \sum_{m=-\infty}^{\infty} \overline{f[m]}g[m+n]$$

INPUT - EXPONENTIAL DISTRIBUTION



BEAM SPLITTER OPERATOR

$$\begin{bmatrix} E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} \mathcal{R}_{31} & \mathcal{T}_{32} \\ \mathcal{T}_{41} & \mathcal{R}_{42} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

- This matrix is called the beam splitter matrix. R and T respectively denote the reflectance and the transmittance of the splitter.
- Equating the incoming energy with the outgoing energy and then the real and the imaginary parts, we get

  $$\mathcal{R}_{31} = -\mathcal{R}_{42} = |\mathcal{R}| \quad \text{and} \quad \mathcal{T}_{32} = \mathcal{T}_{41} = |\mathcal{T}|$$

  which makes the matrix unitary
- The hamiltonian of the system comes out to be

  $$\widehat{H} = \tfrac{1}{2}(\widehat{p}^{\,2} + w^{2}\widehat{q}^{\,2})$$

Thus we can define our creation and annihilation operators as

$$\hat{a} = (2\hbar\omega)^{-1/2}(\omega\hat{q} + i\hat{p})$$
$$\hat{a}^{\dagger} = (2\hbar\omega)^{-1/2}(\omega\hat{q} - i\hat{p})$$

For an electric field of the form $\mathcal{E}(t) = 2\mathcal{E}_0\, \hat{a}\, e^{i\phi}$ the **operator B** comes out to be

$$< pq \mid B \mid mn > \; = \; \frac{f^{(p)}(0)}{p!\sqrt{m!n!}}\delta_{p+q,m+n}\sqrt{p!q!}$$

where

$$f(x) = (T + iRx)^{n}(Tx + iR)^{m}$$

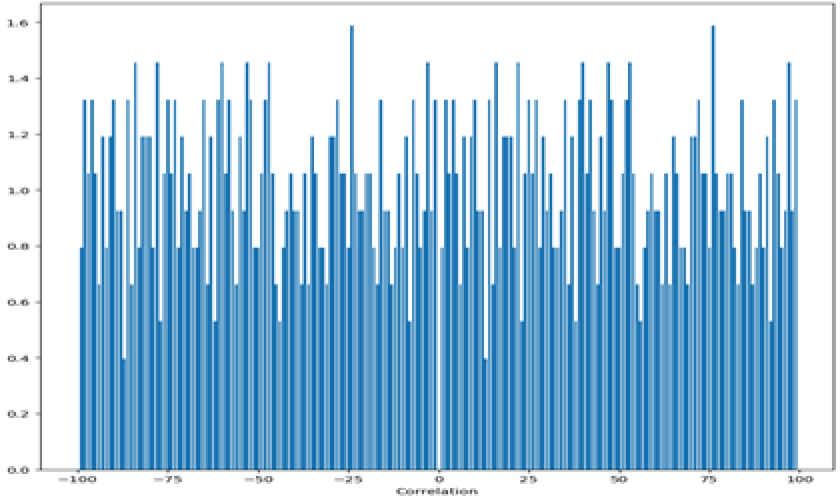
Ref: Quantum Theory Of Light, Gerry and Knight

ONE SINGLE PHOTON INPUT

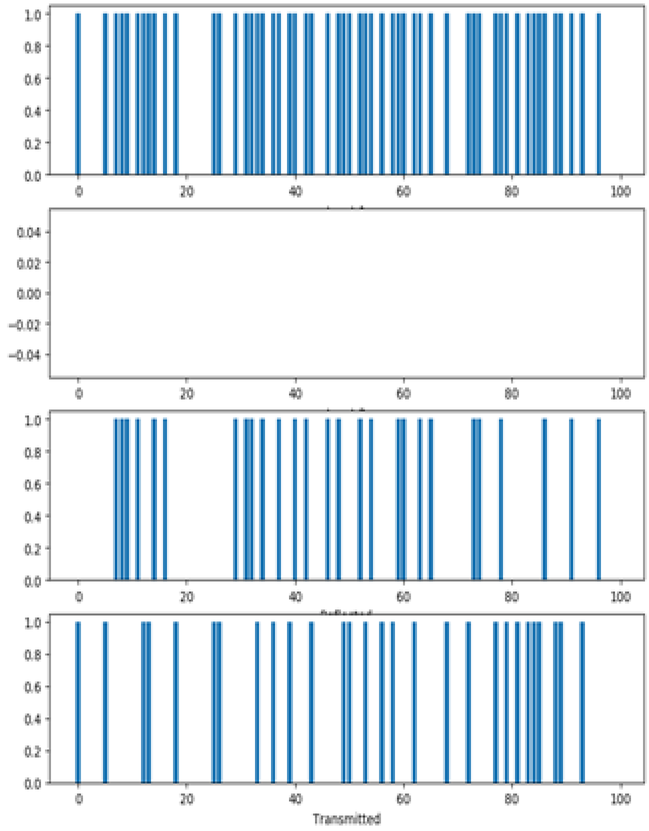
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \\ 0 \end{pmatrix}$$

- Output as seen previously
- Equal probability for |0,1> and |1,0>
- Both states out of phase
- **Antibunching**

CORRELATION



INPUT - UNIFORM DISTRIBUTION

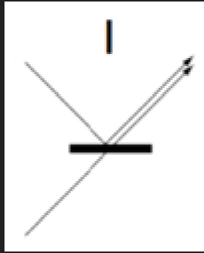


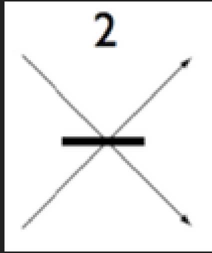
TWO SINGLE PHOTON INPUTS - HONG-OU-MANDEL EFFECT

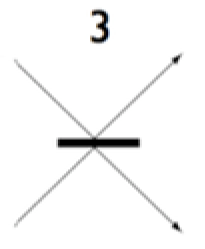
Ref: Wikipedia

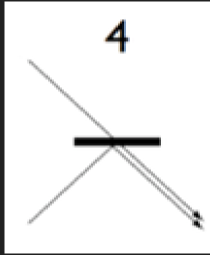
Two photons may enter together

- Four possibilities
- Excitation is possible





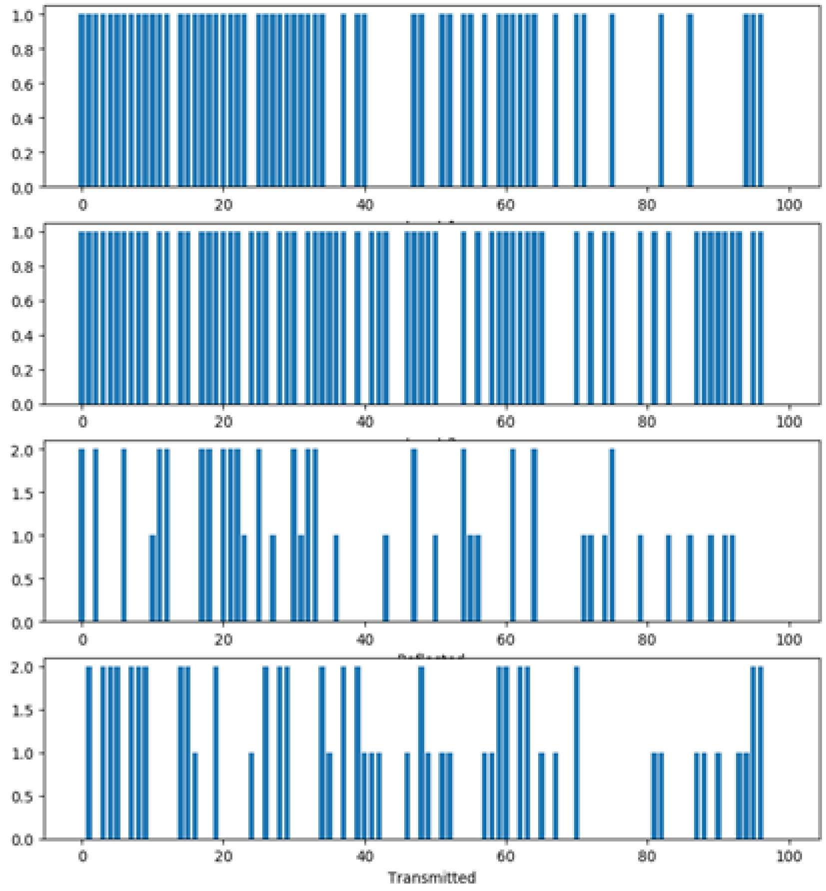




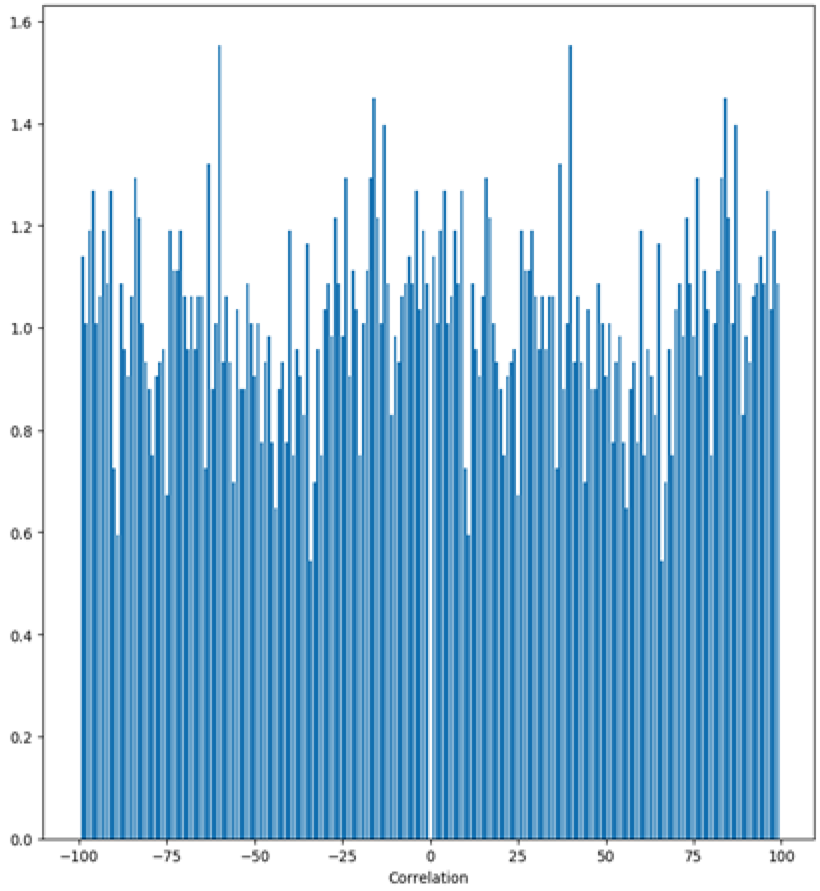
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{sqrt(2)} & 0 & \frac{i}{sqrt(2)} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{i}{sqrt(2)} & 0 & \frac{-1}{2} & 0 & 0 \\ 0 & \frac{i}{sqrt(2)} & 0 & \frac{1}{sqrt(2)} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{i}{sqrt(2)} & 0 & 0 & 0 & \frac{i}{sqrt(2)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{2sqrt(2)} & 0 & \frac{i}{2sqrt(2)} & 0 \\ 0 & 0 & \frac{-1}{2} & 0 & \frac{i}{sqrt(2)} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{2sqrt(2)} & 0 & \frac{-i}{2sqrt(2)} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{2} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{i}{sqrt(2)} \\ 0 \\ 0 \\ 0 \\ \frac{i}{sqrt(2)} \\ 0 \\ 0 \end{pmatrix}$$

- Possibilities 2 and 3 are never actually observed
- Their probabilities are zero
- Both photons will always exit in the same mode.
- Outputs **antibunched**

INPUT - EXPONENTIAL DISTRIBUTION



CORRELATION

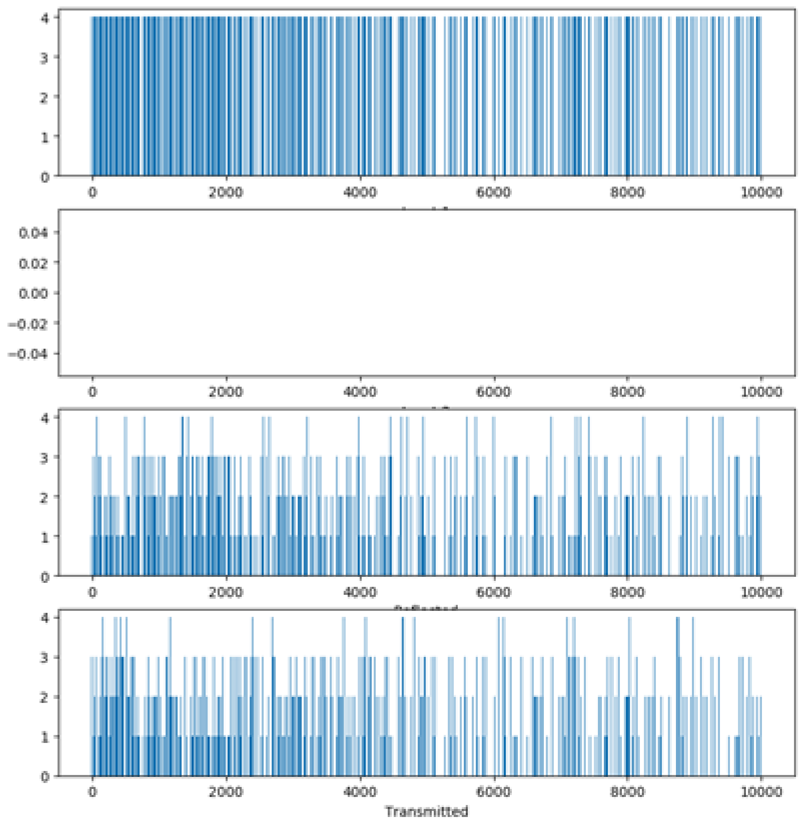


n-PHOTON INPUT

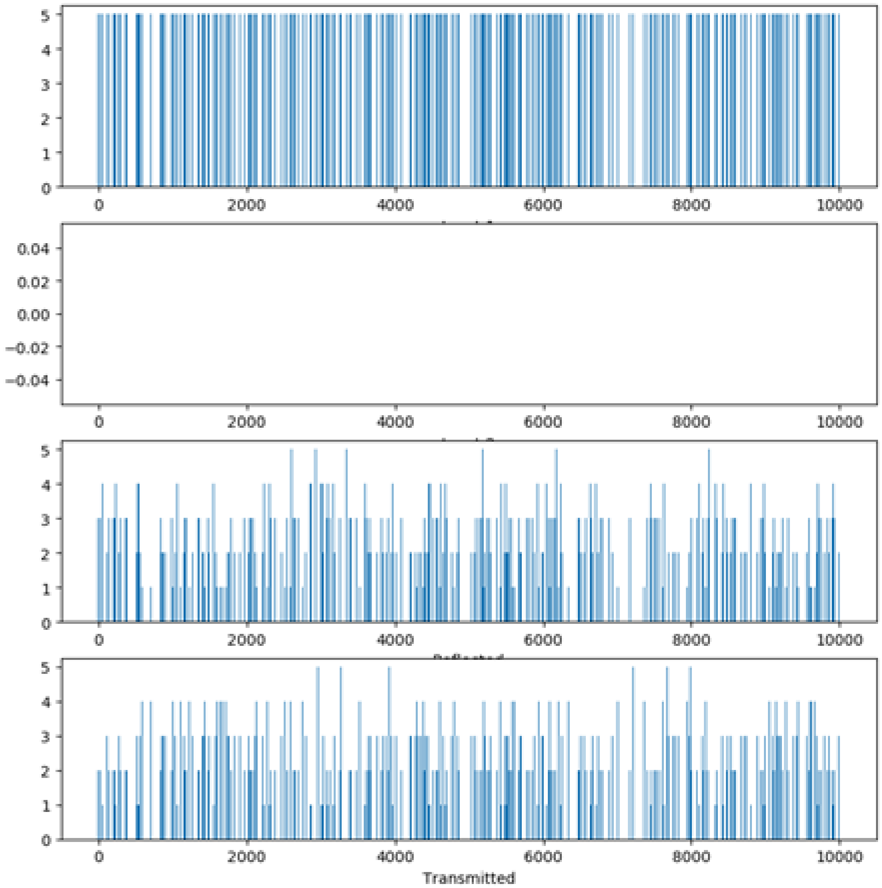
- n photons can leave through either output in n ways
    - Probability depends on operator elements
    - Turns out to be **Poisson** distribution
- No. of photons is conserved
- Evidently **antibunching** absent

$$< pq \mid B \mid mn > \;=\; \frac{f^{(p)}(0)}{p!\sqrt{m!n!}}\delta_{p+q,m+n}\sqrt{p!q!}$$

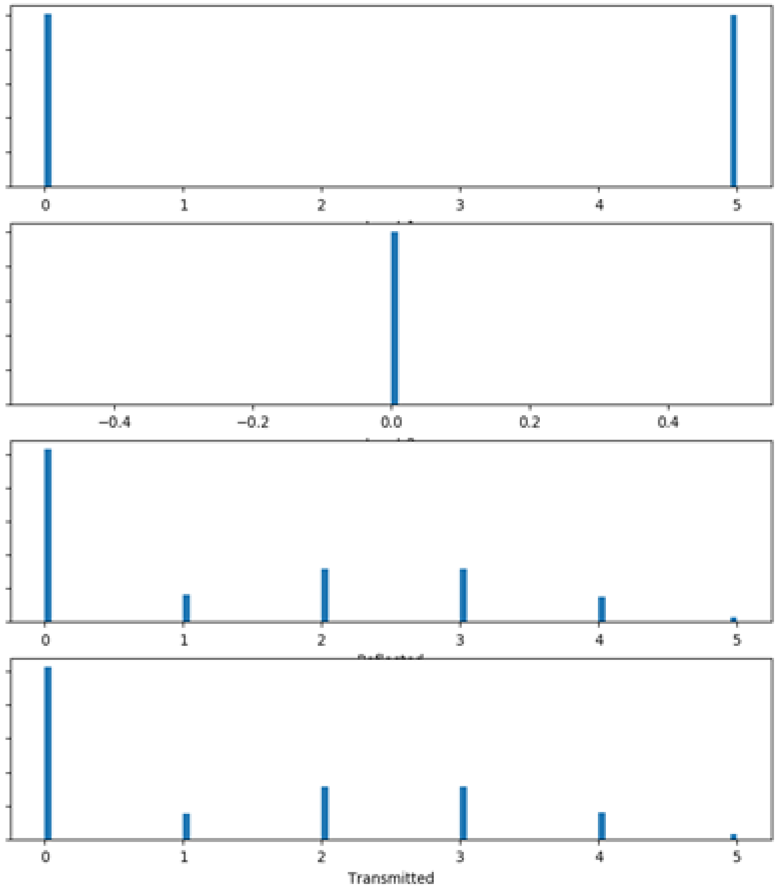
INPUT - UNIFORM DISTRIBUTION, n = 4



INPUT - UNIFORM DISTRIBUTION, n = 5



HISTOGRAM



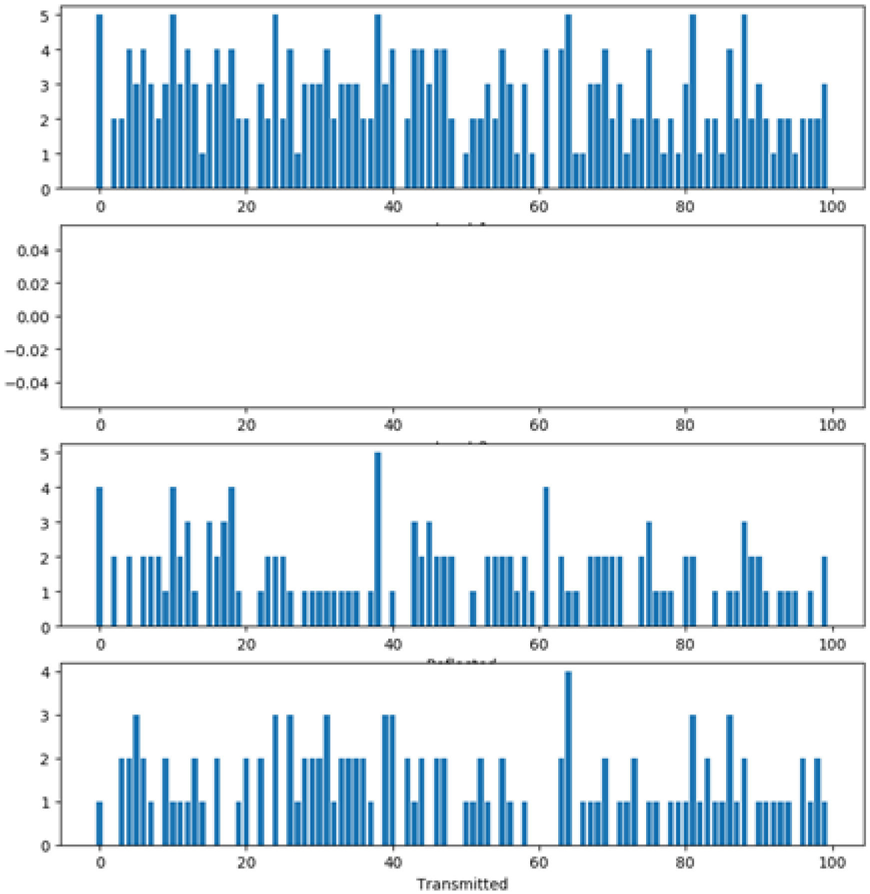
COHERENT STATE INPUT

- Laser model
- Mixture of all eigenstates
  - Poisson probability distribution
- Outputs have Poisson distribution
  - Peak is shifted to a lower state
- No **antibunching** observed
  - Uniform second-order correlation

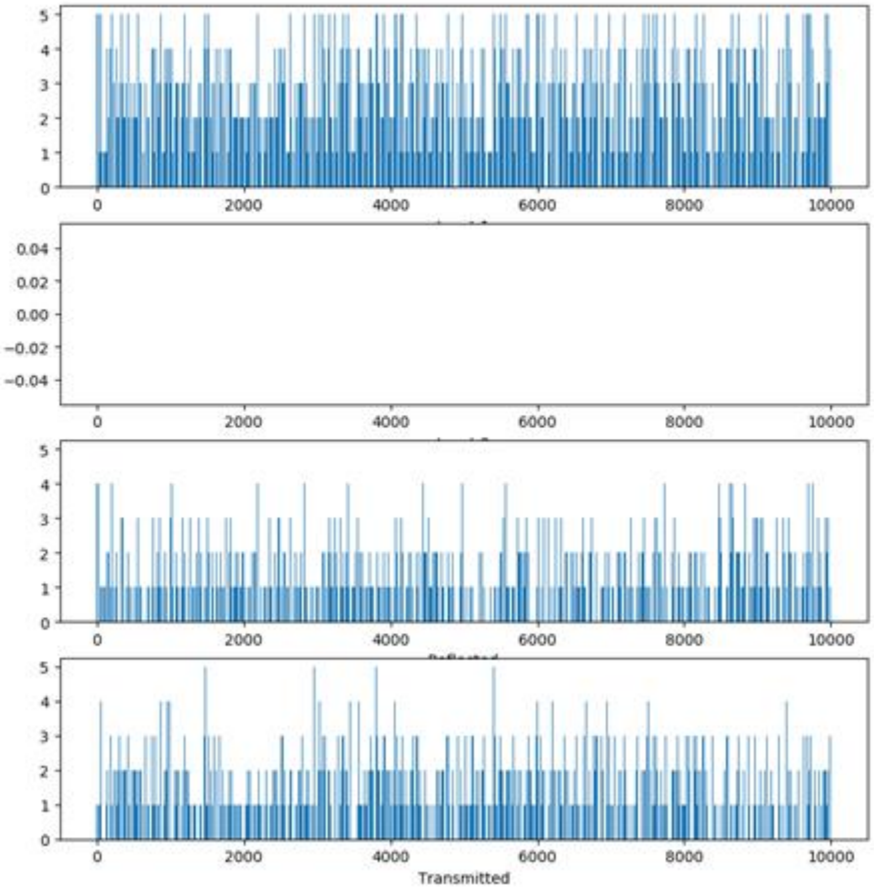
$$\psi_N(x,t) = \sum_{n=0}^{\infty} c_{N,n} \exp\left\{-i\left(n+\frac{1}{2}\right)\omega t\right\} u_n(x)$$

$$\left|c_{N,n}\right|^2 = \frac{N^n \exp(-N)}{n!}$$

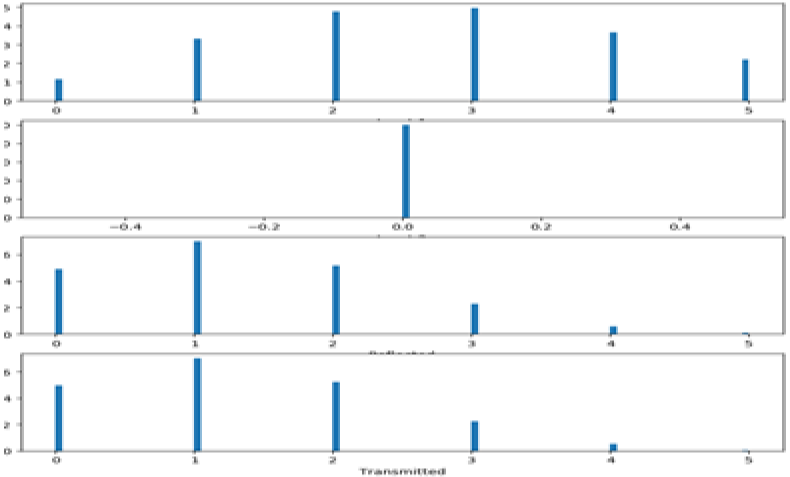
6 STATES ALLOWED, N = 3



6 STATES ALLOWED, N = 3



HISTOGRAM



CORRELATION